IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Applicant:	Simo Mäenpää
Title:	TREADMILL ARRANGEMENT
Serial Number:	09/894,803
Filing Date:	June 29, 2001
Examiner / Unit:	Stephen Crow / 3764
Attorney Docket Number:	TU2X-1-1029

Simo Mäenpää hereby declares as follows:

1. I am the Declarant herein, have personal knowledge of facts contained in this declaration and am competent to testify to the same.
2. I received a Master of Science in Electrical Engineering in 1990 from Helsinki University of technology, where my area of concentration was electrical power engineering including power system engineering, product development (electrical equipment) and electromechanics. In addition to that I have taken some courses of post-graduated studies. During my subject studies in the University I gained extensive basic expertise of electromagnetic field theory. Please, see attached my Diploma of M.Sc. and CV.
3. I am aware of the level of ordinary skill in art relating to receivers of the type shown and described in the subject application (especially relating to the area of theory of the electromagnet fields - the key area of the invention) and I am aware of the relevant prior art which one of ordinary skill would be aware of. It is my opinion that the disclosure relating to these receivers in subject specification is sufficient to enable one of ordinary skill in that art to make and use the subject invention.
4. The following publications show that structures permitting such receivers to recognize different of varying strains of an electromagnetic signal (field) are well known in relevant prior art, e.g. Cheng, D., Field and Wave Electromagnetics. Addison-Wesley Publishing Company, New York, 1991. Other documents concerning the studied field of the invention are listed in the list of the references in the M.Sc. thesis of Pasi Mattila (attached).
5. Being warned that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such will false statements may jeopardize the validity of the subject application or document or any registration resulting therefrom, I-declare that all statements made of my own knowledge are true and all statements made on information and belief are believed to be true.

Dated this 30th of September, 2003

By ______________________

Simo Mäenpää
Declarant

Translated from the relevant parts of the original Finnish:

UNIVERSITY OF TECHNOLOGY

Simo Jaakko Mäenpää
personal identity number 040565-167S (born May 4, 1965),
registered at the University of Technology on July 27, 1984

has taken the degree of

MASTER OF SCIENCE IN ENGINEERING

in accordance with the degree programme in
Electrical Engineering
and has received the grades listed in this certificate
and been granted the title of
MASTER OF SCIENCE IN ENGINEERING

Otaniemi, May 28, 1990

(illegible signature)
Rector

(illegible signature)
Head of Department

(seal of the University of Technology)

(KOPION OIKEAKSI TODISTAVAT)

Degree programme	ELECTRICAL ENGINEERING	
Area of concentration	ELECTRICAL POWER ENGINEERING	

	EXTENT	OVERALL GRADE
GENERAL STUDIES	30.5 cr	very good
SUBJECT STUDIES	98.0 cr	good
ADVANCED STUDIES, SUBJECTS OF SPECIALIZATION		
Power systems engineering	13.5 cr	good
Product development, electrical equipment	13.0 cr	excellent
Electromechanics	14.0 cr	very good
MASTER'S THESIS	20.0 cr	very good
PRACTICAL TRAINING	5.0 cr	
OTHER COURSES	3.0 cr	
TOTAL CREDITS FOR THE DEGREE	189.0 cr	

MASTER'S THESIS "Development of a sales support programme for asynchronous motors"

WAS PREPARED ON THE SPECIAL SUBJECT OF Product development, electrical equipment
UNDER THE SUPERVISION OF Professor Tapani Jokinen,
AND UNDER THE GUIDANCE OF Matti Turtiainen, Master of Science (Abb Strömberg Drives Oy)

THE GRADUATE, WHO RECEIVED HIS LEAVING CERTIFICATE FROM A FINNISH-LANGUAGE EDUCATIONAL ESTABLISHMENT, HAS COMPLETED HIS PROFICIENCY TEST IN FINNISH AND PASSED THE EXAMINATION IN THE SECOND OFFICIAL LANGUAGE, SWEDISH1)

THE STUDY MODULES TAKEN ARE SHOWN IN THE ATTACHED EXTRACT FROM THE REGISTER OF STUDIES.

According to the 1979 University of Technology Degree Rules one credit represents 40 hours of effective study. Three weeks' practical training represents one credit. The minimum number of credits for the degree is 180, of which the thesis comprises 20. In the basic degrees, the grades are excellent (5), very good (4), good (3), very satisfactory (2), satisfactory (2) and fail (0). The overall grades have been calculated by weighting the course grades with the number of credits. Courses can also be graded pass or fail.

1) This language test demonstrates the proficiency in the second official language required of civil servants with a university degree working in a bilingual area under Section 1 of the Act on language proficiency required of civil servants (442/87, section 13, paragraph 1 and section 14, paragraph 1).

For a true translation:
Helsinki, August 16, 1994

VIRALLINEN KÄÄNTÄJÄ AUTHORIZED TRANSLATOR DIANA C. TULLBERG B.A.

Diana Tullberg

Antti Suontausta

Tapani Suontausta

Translated from the relevant parts of the original Finnish:

HELSINKI UNIVERSITY OF TECHNOLOGY EXTRACT FROM THE REGISTER OF STUDIES

Simo Jaakko M ä e n p ä ä
personal identity number 040565-167S (born May 4, 1965),
registered with the University of Technology on July 27, 1984, has completed the study modules listed below:

Code and name of study module		Extent in credits	Grade	Teacher
GENERAL STUDIES:				
0.00.101	Orientation course for new students	0.5	pass	Koskiala
0.01.100	Analytical geometry A	2.5	4	Rikkonen
0.03.122	Physics I	4.5	5	Tuomi
0.07.105	Economics I , basic course	2	4	Jaskari
0.07.110	Economics II, advanced course	2	4	Jaskari
Mat-7.115	Economics III, foreign exchange	2	3	Jaskari
Kie-98.002	Second official language (satisfactory)	1	pass	Katajamäki
Kie-98.102	Technical English reading comprehension	2	pass	Lehtisalo
Kie-98.103	English for everyday use 1	2	3	Benson
0.98.121	German , basic course, 2	2	3	Manner
Tkl-38.100	Principles in communication and research	2	pass	Rahko
3.22.105	Basic industrial economics	3	3	Hankipohja
3.41.131	Engineering drawing	2	2	Pere
3.76.100	Introduction to programming	2	5	Saikkonen
Puu-23.171	Environmental protection, basic course	1	3	Dyer
SUBJECT STUDIES				
0.01.012	Series A	1.5	1	Segercrantz
Mat-1.016	Numerical analysis A	3	2	Piila
0.01.102	Differential calculus A	2	3	Rikkonen
0.01.104	Integral calculus A	2	5	Rikkonen
0.01.106	Differential calculus in several variables A	1.5	4	Rikkonen
0.01.108	Integral calculus in several variables A	1.5	1	Rikkonen
0.01.110	Differential equations A	1	3	Ilkka
0.01.115	Matrix calculus	2.5	3	Kivelä
0.01.118	Theory of functions	2	4	Segercrantz
0.01.120	Integral transforms	2	2	Segercrantz
0.02.100	Introduction to probability A	1.5	2	Koljonen
0.03.123	Physics II	4.5	4	Luomajärvi
0.03.145	Physics III	3	5	Hautojärvi

0.03.146	Physics IV	3	2	Puska
0.03.151	Physics, laboratory course	2	pass	Tuomisaari
Kie-98.165	Technical Swedish I	1	1	Katajamäki
Svt-17.100	Electromechanics	5	3	Luomi
1.17.140	Electrical power technology	2.5	5	Luomi
Svt-18.100	Power systems engineering	5	2	Halonen
Svt-18.151	Power distribution	2.5	2	Mörsky
Svt-18.172	Principles of illumination	2.5	5	Halonen
Svt-18.173	Applications of illumination	2	2	Halonen
1.55.112	Electric circuit analysis	3	4	Valtonen
1.55.113	Electromagnetic fields	3	3	Valtonen
1.55.121	Circuit analysis	2	3	Valtonen
1.55.126	Field theory and basic radio engineering	3	3	Mannersalo
1.66.190	Fundamentals of measurements	3	4	Wallin
1.69.100	Solid state electronics	3	3	Ylilammi
1.72.114	Principles of communications engineering	3	4	Halme
1.74.110	Principles of control engineering	2	2	Niemi
1.79.110	Principles of digital technology	2	3	Rautanen
Svt-81.100	Fundamentals of power electronics	4	4	Mård
1.87.110	Electronics	3.5	4	Porra
Ele-87.120	Electronics, laboratory course	3	3	Porra
Tkl-88.118	Microcomputers, basic course	2	3	Linnavuo
Kon-41.141	Introduction to machine design	2	3	Kivioja
Ene-59.106	Basic energy economics and power plant engineering	3	4	Jahkola
3.76.105	Introduction to data processing	3	5	Bergqvist

ADVANCED STUDIES

Power systems engineering

Svt-18.111	Joint use of power stations	2.5	2	Mörsky
Svt-18.141	Power lines and substations	2.5	3	Naumanen
Svt-18.146	High voltage techniques	2.5	2	Mörsky
Svt-18.156	Seminar on power systems engineering	2.5	4	Mörsky
Svt-18.165	Seminar on the subject of specialization	0.5	3	Mörsky
Svt-18.176	Electric installations in buildings	3	3	Halonen

Product development of electrical equipment

Svt-17.111	Product development	5	5	Jokinen
Svt-17.121	Electrical equipment design	3	4	Saransaari
Svt-17.130	Computer-aided design	2	5	Holmström
Svt-17.170	Electromechanics, special assignment	3	4	Jokinen

Electromechanics

Svt-17.121	Electrical equipment design	3	4	Sarasaari
Svt-17.130	Computer-aided design	2	5	Holmström
Svt-17.153	Numerical methods in electromechanics	3	3	Luomi
Svt-17.161	Electromechanical dynamics	3	2	Perho
Svt-17.170	Electromechanics, special assignment	3	4	Jokinen

OTHER COURSES

Svt-17.194	Postgraduate course in electromechanics	3	4	Eriksson
	Practical training	5	pass	

Otaniemi, May 28, 1990

FACULTY OF CIVIL ENGINEERING

(illegible signature)

Keeper of study register

According to the 1979 University of Technology Degree Rules one credit represents 40 hours of effective study. In the basic degrees, the grades are excellent (5), very good (4), good (3), very satisfactory (2), satisfactory (2) and fail (0). Courses can also be graded pass or fail.

For a true translation:
Helsinki, August 17, 1994

VIRALLINEN KÄÄNTÄJÄ * DIANA C. TULLBERG, B.A. * AUTHORIZED TRANSLATOR

Curriculum Vitae of Simo Mäenpää

Vaalantie 20, 20750 Turku, Finland
+358-2-244 9425, +358-40-5619947

BIOGRAPHICAL DATA

Name:	Simo Jaakko Mäenpää
Date of birth:	4 May, 1965 in Espoo
Marital status:	Married, 3 daughters: Maija (94), Liisa (97), Johanna (00)
Military rank:	Lieutenant, Telecommunication (Military service 85-86)

EDUCATION

2000–2001 Turku School of Economics and Business Administration
- BEI (an Executive MBA-module, 20 credits) Turku, Finland

1998–1999 Turku School of Economics and Business Administration
- JOKO (an Executive MBA-module, 20 credits) Turku, Finland

1990–1994 Helsinki University of Technology (TKK) Espoo, Finland
- Post graduated studies, 31 credits

1984–1990 Helsinki University of Technology (TKK) Espoo, Finland
- Master of Science in Electrical Engineering
- Area of concentration / advanced studies: Electrical power engineering, Power systems engineering, Product development of electrical equipment, Electromechanics
- Master's thesis "Development of a sales support program for asynchronous motors" with grade very good
- Graduated Summa: Very Good, average grade of courses 3,8 (scale 1-5).

1981–1984 Nurmijärven Yhteiskoulu Nurmijärvi, Finland
- Secondary school graduate: 4 Laudatur and 2 Cum Laude in higher school exam, the average grade of the school-leaving certificate: 9,5 (scale 4-10).

Several professional courses and seminars, the latest:
- Modern quality management, IIR, 2002, 2 days (acted as speaker as well)
- Presentation skills, Infor, 2002, 2 days
- Leadership and managerial training (internal company training), Turun ammattikorkeakoulu, 2001-2002, 3 credits, (acted as lecturer as well)
- Managerial training, (internal company tr.), Impulssi Instituutti, 2000, 3 days
- 3D - Efficient leadership, KOLME DEE OY, 1999, 3 days
- Partnership and partnering in business networks and relationships, Turun ammattikorkeakoulu, 1998-1999
- Measurement of efficiency in R&D, Insko 1999, 1 day

PROFESSIONAL EXPERIENCE IN TUNTURI OY LTD

01/11– Tunturi Oy Ltd Turku, Finland

Research and Quality Director
- Strategic and operational work in international business environment being responsible for company's research and technology issues concerning fitness equipment and quality development in the company.
- Monitoring and creating new business concepts and opportunities based on new technologies. Managing strategic developing and research projects.

- Developing processes, systems and tools in the company's business network.

00/11–01/10 Tunturi Oy Ltd Turku, Finland

Product Development and Quality Director

- As above but the main focus on managing product development projects.

99/01–00/10 Tunturi Oy Ltd Turku, Finland

Research and Product development Manager (Fitness Division)

- Developing and implementing a new "modus operandi" in R&D to manage product development and projects in the networks of companies.
- Improving project and product documentation: the PDM – system (Aton/MST9000) was specified and implemented.
- Developing a systematic product road map – concept and working model.
- Head of the Customer Focus Management –organisation.
- Resulting essential improvement in time-to-market performance and in accuracy of the project execution.

97/10–98/12 Tunturipyörä Oy Turku, Finland

Customer Focus Manager

- Establishing a Customer Focus Management - function for the company including after sales-, warranty-issues, technical support and training, competitive intelligence system, tools for handling customers' complaints and company's internet/extranet issues and development.
- Supervising the large spare part relocation project from Seattle to Toronto.
- Direct warranty expenses were reduced over 50% within 2 years. Customer satisfaction increased due to improved service and product quality.

PROFESSIONAL EXPERIENCE IN ABB COMPANIES

96/10–97/09 ABB Industry Oy, Induction Machines Helsinki, Finland

Quality Manager, Customer Focus Manager

- A member in the management group of Induction Machines (300 employees) and in the quality management group of Machines Division (800 employees).
- Responsible for the overall development of customer satisfaction and quality. Supervising the after sales department.
- Owner of customer complaint resolution process (CCRP) in Induction Machines. Resolving the most demanding customer complaints and inquiries.
- Developing PC-based tools for evaluating and analysing quality expenses and customer satisfaction.

94/10–96/09 ABB Industrial Systems Inc. New Berlin, WI, USA

Product Manager, Manager Applications (Large AC Machines)

- Managing the product related activities with regards to the technical issues of the large AC motors.
- Supervising the group of application engineers to ensure the meeting of the goals for sales volume and profitability.
- Developing and making product and application presentations to customers and acting as a technical resource and resolving customer inquiries.
- Directly responsible for proposal process and developing sales management

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automation PC-tools. The proposal process automation PC-tools resulted in more than a 20% improvement in productivity.

- My professional and management skills as well as my knowledge in PCs and programming were considered excellent (ref. performance evaluation).

93/08–94/10 ABB Industry Oy, HX-Machines Helsinki, Finland

Product Manager (R&D), HX-Machines

- Sales and marketing of AC motors for the oil and gas – business segment.
- Responsible for developing and designing motor drives for other demanding applications (e.g. azibod-motors).
- Supervising research applied for technical calculations.

92/08–93/08 ABB Industry Oy, HX-Machines Helsinki, Finland

Project Manager, Troll-offshore and oil rig project

- Responsible for developing the very first adjustable speed AC-motor-drive for offshore drilling. Managing the project execution.
- ABB has delivered several similar type of drilling motors world wide after the Troll project.

90/05–92/08 ABB Strömberg Drives Oy Helsinki, Finland

Area Sales Manager, Technical support in Sales (Asynchr. Machines)

- Responsible for internal sales in ABB Strömberg Drives Oy including customer meetings, presentations and sales negotiations.
- Technical sales support, training and documentation. Selecting and dimensioning motors according to customers' specifications.
- Continuing in developing the sales support PC-program. Distributing it to sales organisation abroad and providing user training. The sales tool became the official sales tool for large AC machines in ABB.

89/05–90/05 ABB Strömberg Drives Oy Helsinki, Finland

Master's thesis worker

- Research work for my master's thesis "Sales supporting expert system for asynchronous machines"
- Developing, designing and programming the PC based sales tool for sales support and electromagnetic dimensioning of large AC machines.

PROFESSIONAL EXPERIENCE IN OTHER COMPANIES

1987–1989 Helsinki University of Technology (TKK) Espoo, Finland

***Laboratorium assistant, research assistant** (the Laboratorium of Electromechanics)*

- Laboratorium assistant in "Electrical power engineering"-course guiding students in their lab-works.
- Research assistant studying the usage of solar cells in electrical cars.

1984–1988 Work on temporary basis during my university years

- Act as a substitute for maths, physics and chemistry teachers, several occasions in Nurmijärvi Yhteiskoulu.
- Engineer-Trainee, Sähkösuunnittelutoimisto Mauno Ahonen Oy (the summer 87)

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- Journalist-trainee, Nurmijärven Sanomat (the summer 86)
- Industrial worker: Oy Alko Ab (Rajamäki Bottling factory, the summer 84), Kahi-Tiili Oy (the summer 86), Electric plant of Nurmijärvi (the summer 88)

LINGVISTIC SKILLS

Finnish	mother tongue
English	very good
Swedish	'out of tune' - used to be fair
German	'out of tune' - used to be fair

SPECIAL SKILLS AND KNOW-HOW

- Ability to develop and re-engineer procedures and processes
- Versatile know-how about the most common PC-SW and programming
- Standardization work (Member of CEN TC 136 / WG 4)

INTERESTS

Family-life, redecorating, sports in many forms (tennis, badminton, jogging, weight lifting), investment activities (e.g. stocks), new technology (e.g. PC-multimedia, audio-visual systems), music (playing and listening), Vaalan omakotiyhdistys

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SIMO MÄENPÄÄ
02-
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Second Edition

Field and Wave Electromagnetics

David K. Cheng
Life Fellow, I.E.E.E.;
Fellow, I.E.E.; C. Eng.

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Preface

The many books on introductory electromagnetics can be roughly divided into two main groups. The first group takes the traditional development: starting with the experimental laws, generalizing them in steps, and finally synthesizing them in the form of Maxwell's equations. This is an inductive approach. The second group takes the axiomatic development: starting with Maxwell's equations, identifying each with the appropriate experimental law, and specializing the general equations to static and time-varying situations for analysis. This is a deductive approach. A few books begin with a treatment of the special theory of relativity and develop all of electromagnetic theory from Coulomb's law of force; but this approach requires the discussion and understanding of the special theory of relativity first and is perhaps best suited for a course at an advanced level.

Proponents of the traditional development argue that it is the way electromagnetic theory was unraveled historically (from special experimental laws to Maxwell's equations), and that it is easier for the students to follow than the other methods. I feel, however, that the way a body of knowledge was unraveled is not necessarily the best way to teach the subject to students. The topics tend to be fragmented and cannot take full advantage of the conciseness of vector calculus. Students are puzzled at, and often form a mental block to, the subsequent introduction of gradient, divergence, and curl operations. As a process for formulating an electromagnetic model, this approach lacks cohesiveness and elegance.

The axiomatic development usually begins with the set of four Maxwell's equations, either in differential or in integral form, as fundamental postulates. These are equations of considerable complexity and are difficult to master. They are likely to cause consternation and resistance in students who are hit with all of them at the beginning of a book. Alert students will wonder about the meaning of the field vectors and about the necessity and sufficiency of these general equations. At the initial stage students tend to be confused about the concepts of the electromagnetic model, and they are not yet comfortable with the associated mathematical manipulations. In any case, the general Maxwell's equations are soon simplified to apply to static fields,

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Equation (6–23) enables us to find the vector magnetic potential **A** from the volume current density **J**. The magnetic flux density **B** can then be obtained from $\nabla \times \mathbf{A}$ by differentiation, in a way similar to that of obtaining the static electric field **E** from $-\nabla V$.

Vector potential **A** relates to the magnetic flux Φ through a given area S that is bounded by contour C in a simple way:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}. \tag{6–24}$$

The SI unit for magnetic flux is weber (Wb), which is equivalent to tesla-square meter ($\text{T}\cdot\text{m}^2$). Using Eq. (6–15) and Stokes's theorem, we have

$$\Phi = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} \qquad \text{(Wb)}. \tag{6–25}$$

Thus, vector magnetic potential **A** does have physical significance in that its line integral around any closed path equals the total magnetic flux passing through the area enclosed by the path.

6–4 The Biot-Savart Law and Applications

In many applications we are interested in determining the magnetic field due to a current-carrying circuit. For a thin wire with cross-sectional area S, dv' equals $S\,d\ell'$, and the current flow is entirely along the wire. We have

$$\mathbf{J}\,dv' = JS\,d\boldsymbol{\ell}' = I\,d\boldsymbol{\ell}', \tag{6–26}$$

and Eq. (6–23) becomes

$$\boxed{\mathbf{A} = \frac{\mu_0 I}{4\pi} \oint_{C'} \frac{d\boldsymbol{\ell}'}{R} \qquad \text{(Wb/m)},} \tag{6–27}$$

where a circle has been put on the integral sign because the current I must flow in a closed path,[†] which is designated C'. The magnetic flux density is then

$$\begin{aligned} \mathbf{B} = \nabla \times \mathbf{A} &= \nabla \times \left[\frac{\mu_0 I}{4\pi} \oint_{C'} \frac{d\boldsymbol{\ell}'}{R}\right] \\ &= \frac{\mu_0 I}{4\pi} \oint_{C'} \nabla \times \left(\frac{d\boldsymbol{\ell}'}{R}\right). \end{aligned} \tag{6–28}$$

[†] We are now dealing with direct (non-time-varying) currents that give rise to steady magnetic fields. Circuits containing time-varying sources may send time-varying currents along an open wire and deposit charges at its ends. Antennas are examples.

It is very important to note in Eq. (6–28) that the *unprimed* curl operation implies differentiations with respect to the space coordinates of the *field point*, and that the integral operation is with respect to the *primed source coordinates*. The integrand in Eq. (6–28) can be expanded into two terms by using the following identity (see Problem P.2–37):

$$\nabla \times (f\mathbf{G}) = f\nabla \times \mathbf{G} + (\nabla f) \times \mathbf{G}. \tag{6–29}$$

We have, with $f = 1/R$ and $\mathbf{G} = d\boldsymbol{\ell}'$,

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\oint_{C'}\left[\frac{1}{R}\nabla \times d\boldsymbol{\ell}' + \left(\nabla\frac{1}{R}\right) \times d\boldsymbol{\ell}'\right]. \tag{6–30}$$

Now, since the unprimed and primed coordinates are independent, $\nabla \times d\boldsymbol{\ell}'$ equals 0, and the first term on the right side of Eq. (6–30) vanishes. The distance R is measured from $d\boldsymbol{\ell}'$ at (x', y', z') to the field point at (x, y, z). Thus we have

$$\begin{aligned}
\frac{1}{R} &= [(x-x')^2 + (y-y')^2 + (z-z')^2]^{-1/2}; \\
\nabla\left(\frac{1}{R}\right) &= \mathbf{a}_x\frac{\partial}{\partial x}\left(\frac{1}{R}\right) + \mathbf{a}_y\frac{\partial}{\partial y}\left(\frac{1}{R}\right) + \mathbf{a}_z\frac{\partial}{\partial z}\left(\frac{1}{R}\right) \\
&= -\frac{\mathbf{a}_x(x-x') + \mathbf{a}_y(y-y') + \mathbf{a}_z(z-z')}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}} \\
&= -\frac{\mathbf{R}}{R^3} = -\mathbf{a}_R\frac{1}{R^2},
\end{aligned} \tag{6–31}$$

where $\mathbf{a}_R$ is the unit vector directed *from the source point to the field point*. Substituting Eq. (6–31) in Eq. (6–30), we get

$$\boxed{\mathbf{B} = \frac{\mu_0 I}{4\pi}\oint_{C'}\frac{d\boldsymbol{\ell}' \times \mathbf{a}_R}{R^2} \quad \text{(T)}.} \tag{6–32}$$

Equation (6–32) is known as ***Biot-Savart law***. It is a formula for determining $\mathbf{B}$ caused by a current I in a closed path C' and is obtained by taking the curl of $\mathbf{A}$ in Eq. (6–27). Sometimes it is convenient to write Eq. (6–32) in two steps:

$$\boxed{\mathbf{B} = \oint_{C'} d\mathbf{B} \quad \text{(T)},} \tag{6–33a}$$

with

$$\boxed{d\mathbf{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\boldsymbol{\ell}' \times \mathbf{a}_R}{R^2}\right) \quad \text{(T)},} \tag{6–33b}$$

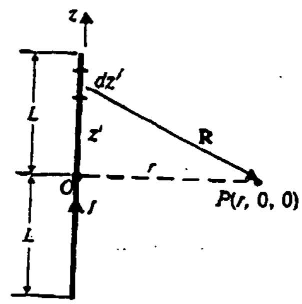

FIGURE 6-5
A current-carrying straight wire (Example 6-4).

which is the magnetic flux density due to a current element $I\,d\ell'$. An alternative and sometimes more convenient form for Eq. (6-33b) is

$$\boxed{d\mathbf{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\boldsymbol{\ell}' \times \mathbf{R}}{R^3}\right) \qquad \text{(T)}.} \tag{6-33c}$$

Comparison of Eq. (6-32) with Eq. (6-10) will reveal that Biot-Savart law is, in general, more difficult to apply than Ampère's circuital law. However, Ampère's circuital law is not useful for determining **B** from I in a circuit if a closed path cannot be found over which **B** has a constant magnitude.

EXAMPLE 6-4 A direct current I flows in a straight wire of length $2L$. Find the magnetic flux density **B** at a point located at a distance r from the wire in the bisecting plane: (a) by determining the vector magnetic potential **A** first, and (b) by applying Biot-Savart law.

Solution Currents exist only in closed circuits. Hence the wire in the present problem must be a part of a current-carrying loop with several straight sides. Since we do not know the rest of the circuit, Ampère's circuital law cannot be used to advantage. Refer to Fig. 6-5. The current-carrying line segment is aligned with the z-axis. A typical element on the wire is

$$d\boldsymbol{\ell}' = \mathbf{a}_z\, dz'.$$

The cylindrical coordinates of the field point P are $(r, 0, 0)$.

a) *By finding* **B** *from* $\nabla \times \mathbf{A}$. Substituting $R = \sqrt{z'^2 + r^2}$ into Eq. (6-27), we have

$$\begin{aligned}\mathbf{A} &= \mathbf{a}_z \frac{\mu_0 I}{4\pi}\int_{-L}^{L} \frac{dz'}{\sqrt{z'^2 + r^2}} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi}\left[\ln\left(z' + \sqrt{z'^2 + r^2}\right)\right]\Big|_{-L}^{L} \\ &= \mathbf{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{L^2 + r^2} + L}{\sqrt{L^2 + r^2} - L}.\end{aligned} \tag{6-34}$$

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